**Experiment No:** M1

**Experiment Name:** Free Fall and Atwood's Machine

**Objective:** Determining the gravitational acceleration using a free falling object and an Atwood's machine. Investigating the position vs. time, velocity vs. time relations for the two systems.

**Keywords:** Inertia, Moment of Inertia, Torque, Angular Velocity, Angular Momentum.

## Theoretical Information:

### Gravitational acceleration and free fall

Newton's law of universal gravitation dictates that two point particles placed at a distance will attract each other with a force that is proportional to their masses and inversely proportional to the square of the distance between them. The law also states that this force lies on the line that connects two points and is directed towards the other body.

Its magnitude can be expressed by the following equation:

$$F = G\frac{m_1 m_2}{r^2} \tag{1.1}$$

Here $m_1$ and $m_2$ represents the masses of point particles while $r$ represents the distance between them. $G$ is called the universal gravitational constant.

Newton has also proved a theorem which states that a thin spherical shell of uniform mass distribution gravitationally interacts with its outer region like a point particle as if its total mass is concentrated at its center. (The second part of the theorem states that the shell doesn't interact with its interior region at all because of the cancellation of the forces.) This theorem is called the *spherical shell theorem*. According to one rumor Newton waited several years to publish his gravitational law because he wanted to find a legitimate proof to this theorem.

Now let us consider the motion of bodies on Earth's surface using Newton's gravitational law and the spherical shell theorem. The objects that we encounter everyday are very small compared to the Earth's size so they can be treated as point particles with respect to Earth. Furthermore, the spherical shell theorem allows us to treat the Earth itself as a point particle given that its approximate shape is close to a sphere. So it is very reasonable to use equation 1.1 directly. The gravitational force on a body near Earth's surface can then be calculated by:

$$F = G\frac{Mm}{R^2} \tag{1.2}$$

Here $M$ represents the mass of the Earth, $R$ stands for its radius and $m$ is the mass of the body. If the only force on the body is the gravitational force then we can substitute this to Newton's second law of motion and write the equation of the motion as follows:

$$G\frac{Mm}{R^2} = m'a \tag{1.3}$$

In this equation while $m$ represents the body's *gravitational mass*, $m'$ stands for its *inertial mass*. If we use the *equivalence principle* which states that these two quantities can be regarded as equal we can cancel out the mass of the body and we get the following expression 1.4 for the gravitational acceleration:

$$a = G\frac{M}{R^2} \tag{1.4}$$

As one can see the gravitational acceleration near Earth's surface depends on the mass and the radius of Earth. If we substitute the SI values of *G*, *M* and *R* to equation 1.4 we get a value of 9,80 m/s$^2$ for the gravitational acceleration. Note that this quantity (which is generally represented by the symbol *g)* is an approximate value and its exact value depends on the local features (mainly latitude and altitude) of the geographical position on Earth's surface.
Equation 1.5 is called the International Gravity Formula and can be used to estimate the value of gravitational acceleration at a given geographical position on Earth.

$$g = 9{,}780327\times (1 + A\sin^2 L - B\sin^2 2L) - 3.086\times 10^{-6}\times H \tag{1.5}$$

$A$ = 0,0053024
$B$ = 0,0000058
$L$ = Latitude
$H$ = Altitude (from sea level, in units of meters)

In this experiment you will study the motion of a free falling body by measuring the height and the time of the free fall. Since it is a motion with constant acceleration the relationship between those two quantities will be given by the kinematics equations that were derived in the theoretical background of experiment 2. In the experiment there will be no initial velocity so the relation of falling height and time can be written as:

$$h = \frac{1}{2}gt^2 \tag{1.6}$$

**Atwood's Machine**

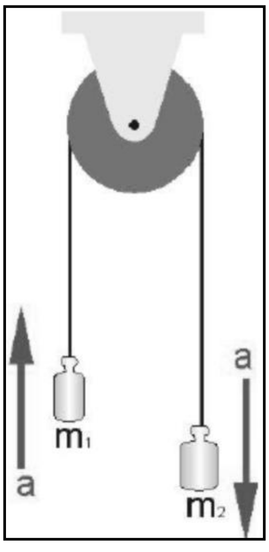


**Figure 1.1** Atwood's machine

Atwood machine is an apparatus invented by English mathematician George Atwood in the year 1784 to study and verify the laws of motion with constant acceleration. It consists of two bodies that move in the horizontal direction which are attached to each other by a string going over a pulley as shown in Figure 1.1. To calculate the accelerations of $m_1$ and $m_2$ we begin by drawing the free body diagrams of two bodies and the pulley as shown in Figure 1.2. $m_2$ is assumed to be greater than $m_1$.

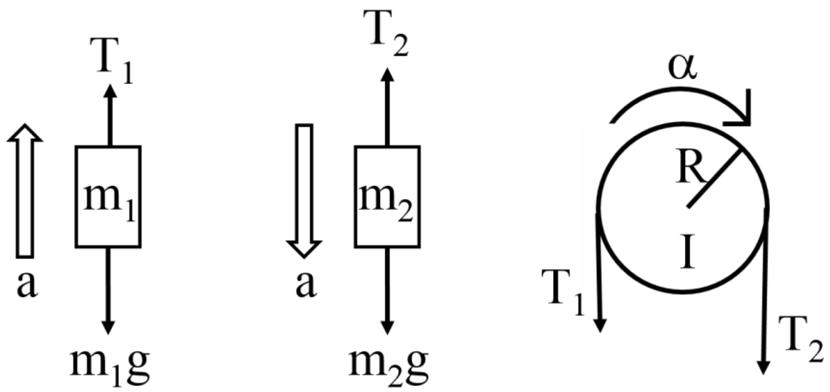


**Figure 1.2** Free body diagrams of $m_1$, $m_2$ and the pulley

The equations of motion of the parts of Atwood's machine can be written as follows:

$$m_1 a = T_1 - m_1 g \tag{1.7}$$

$$m_2 a = m_2 g - T_2 \tag{1.8}$$

$$I\alpha = (T_2 - T_1)R \tag{1.9}$$

While equation 1.7 and equation 1.8 are directly written from Newton's second law of motion equation 1.9 is the application of that law to the rotational motion of the pulley. While the right hand side of Eq. 1.9 represents the net torque acting on the pulley, $I$ term on the left hand side stands for the moment of inertia and α for the angular acceleration of the pulley. We have 3 equations but 4 unknowns which are $T_1$, $T_2$, $a$ and $\alpha$. So we need another equation to solve the unknowns. The fourth equation comes from the observation (or hypothesis) that the rope doesn't "slide" when passing through the pulley which assures that the magnitude of the tangential velocity of the pulley's edge matches with the velocity of the objects. The following relation between the angular and linear accelerations can also be directly derived from this condition since the derivative of the velocities would also equate to each other.

$$\alpha R = a \tag{1.10}$$

Now given that we have four equations we can pull out $T_1$ from Eq. 1.7, $T_2$ from Eq. 1.8, $\alpha$ from Eq. 1.10 and substitute them in Eq. 1.9 to get the following identity:

$$I\frac{a}{R} = (m_2 g - m_2 a - m_1 a - m_1 g)R \tag{1.11}$$

$a$ can be solved from Eq. 1.11 as follows:

$$a = \frac{(m_2 - m_1)g}{\left(m_1 + m_2 + \frac{I}{R^2}\right)} \qquad 1.12$$

ince the pulley has a shape of a disk its moment of inertia would be given by $MR^2/2$. If we substitute this in Eq. 1.12 we get the following expression for the linear acceleration of the system:

$$a = \frac{(m_2 - m_1)g}{\left(m_1 + m_2 + \frac{M}{2}\right)} \qquad 1.13$$

As one can see from Eq. 1.13, the system will exhibit a motion with constant acceleration. The kinematics equations of such a motion is given as follows: (for a comprehensive derivation see the theoretical background of Experiment 2)

$$v = v_0 + at \qquad 1.14$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad 1.15$$